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50 TeV High Field Lattice: Observations from a Golden Cell

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50 TeV high field lattice: observations from a golden cell.

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This short note was written during a two-week "Futures" workshop held at Fermilab in July, 1997. It stresses the obvious point that the first important parameters that must be chosen in designing a high-field VLHC lattice are the phase advance and length of the standard cell. These decisions are not completely trivial and should be made only after some serious work has been done. Separated function and combined function machines are both under consideration.

God, grant me the serenity to accept the things I cannot change,
courage to change the things I can, and the wisdom to know the difference.
— Serenity Prayer
Reinhold Neibuhr

The principal thing that cannot be changed is the Lorentz force law, from which comes the ubiquitous expression relating momentum to magnetic rigidity, $p = eB\rho$. For singly charged particles, this is written in convenient units as follows,

$$B\rho \text{ [T-m]} = 3.33564\ldots \times p \text{ [GeV/c]} \quad .$$

While it is somewhat less immutable, we will consider the specification of building a circular p-p collider at $\sqrt{s} = 100$ TeV using 12.5 Tesla dipoles to be firmly established. From these is obtained the circumference that must be taken up by the arcs.

$$2\pi\rho = C_{\text{arcs}}[\text{km}] = 20.958\ldots \times p[\text{TeV/c}]/B[\text{T}] = 83.834\ldots$$

One reads of a high-field VLHC collider having a circumference of ≈ 100 km, which means that $\approx 20\%$ would not be in the arcs.

Two scenarios are under consideration: the first envisions that the ring will be built up from FODO cells, and the other proposes that it be built using combined function magnets. At this time, neither possibility has been excluded.

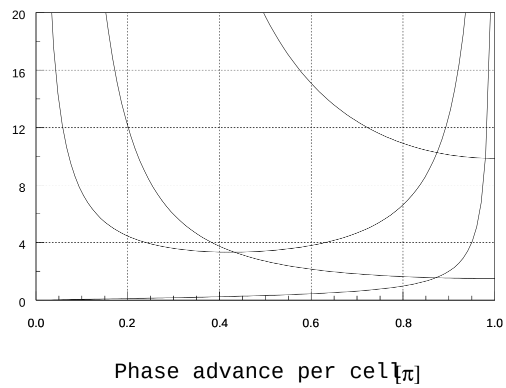

Figure 1: Normalized curves for the FODO model: $\beta_+/L, D_+\rho/L^2$, $N^2\alpha$, and $-\pi\xi/N$.

1 Thin lens, FODO model

The key things that *can* be changed are the length, $2L$, and "phase advance," ψ, of the machine's "standard cell."[1] The thin lens FODO model has the advantage that one can derive a number of simple, analytic expressions relating first order optics parameters to the phase advance through a cell. To begin with, the maximum and minimum horizontal β – which occur at the "center" of the F and D quads, respectively – are given by one of the two equivalent expressions,

$$\beta_\pm/2L = \frac{1 \pm \sin(\psi/2)}{\sin\psi} \tag{1}$$

$$\beta_\pm/2f = \frac{1 \pm \sin(\psi/2)}{\cos(\psi/2)} \tag{2}$$

β_+ from Eq. 1 is plotted as one of the curves in Figure 1. Other scaled parameters of interest, for the same model, include the maximum and minimum dispersion,

$$\rho D_\pm/L^2 = D_\pm/L\theta = \frac{1}{\sin^2(\psi/2)} \pm \frac{1}{2\sin(\psi/2)} \ , \tag{3}$$

the natural chromaticity,

$$\xi/N = -\frac{1}{\pi}\tan(\psi/2) \ ,$$

[1] This number is actually the imaginary part of the log of the eigenvalue of the one-turn map through the cell; that is, $2\pi\nu$ of a standard cell. It literally applies to a machine constructed only from standard cells (and, if you will, perfectly matched straight sections).

and an approximate expression for the momentum compaction,

$$N^2\alpha \approx \left(\frac{\pi}{\sin(\psi/2)}\right)^2 \quad ,$$

where $N = C_{\rm arcs}/2L$ is the number of standard cells. Some of these are also shown in Figure 1. (Derivations can be found in Edwards and Syphers[3] or Bryant and Johnsen[1].)

The minimum value of β_+ occurs at the value of $\sin(\psi/2)$ that satisfies the polynomial,

$$(s+1)(s^2+s-1) = 0$$

The first factor is not physical; the second is solved by the golden ratio,

$$\sin(\psi/2) = r_G = (\sqrt{5}-1)/2 = 0.618034\ldots \tag{4}$$

That it *is* the golden ratio is of no practical importance, but since it is the only original observation in this paper, we will make as much of it as possible. Accordingly, we call the FODO model designed so as to satisfy Eq.(4) a "golden cell," with "golden phase advance" per cell $\psi = 76.345\ldots^\circ$. Of much greater importance is the fact that the curve is rather flat over a large domain, so that minimizing β_+ is not a particularly restrictive constraint.

For the golden cell,

$$\min\beta_+ = (1/r_G^{3/2} + 1/r_G^{1/2}) \times L \approx 3.33019\ldots \times L \quad .$$

We rewrite this in terms of the number of cells in the ring,

$$\begin{aligned} \min\beta_+[\mathrm{m}] &= (3\ 1/3)^2 \frac{\pi}{N} \frac{p[\mathrm{GeV/c}]}{B[\mathrm{T}]} \\ &\approx 140{,}000/N, \end{aligned}$$

where, for aesthetic purposes, we have substituted $(3\ 1/3)^2$ for $3.33564\ldots \times 3.33019\ldots$. A rather extreme upper bound on N can be found by examining its implications for the integrated quadrupole strength. Within the model,

$$B'l = \frac{2}{\pi} BN \sin(\psi/2) = \frac{2r_G}{\pi} BN = 0.39345\ldots \times BN \quad ,$$

where $B'l$ is the integrated quad gradient, B is the dipole field, and N is the number of standard cells in the lattice; that last scaling coefficient applies to the golden cell only. Still assuming that $B = 12.5$ T, and $N \approx 500-1000$, we would need something like ≈ 15 m quadrupoles with operating gradients of $\approx 160-330$ T/m. By comparison, quadrupoles in Tevatron's standard cell have ≈ 78 T/m gradient and 1.7 m length. If we accept this as the upper limit on N, then $\beta_+ \geq 140-280$ m. Let's say, realistically, that we expect $\beta_+ \geq 300$ m from these simple considerations.

Similar expressions provide the values of other parameters in terms of r_G for the golden cell and the scaling parameters, L or, equivalently, N. For example,

$$\begin{aligned} \beta_- &= (1/r_G^{3/2} - 1/r_G^{1/2}) \times L \ = \ 0.78615\ldots \times L \quad , \\ \alpha &\simeq (\pi/r_G)^2/N^2 \ = \ 25.83896\ldots/N^2 \quad , \\ \xi &= -(r_G^{1/2}/\pi) \times N \ = \ -0.25024\ldots \times N \quad . \end{aligned}$$

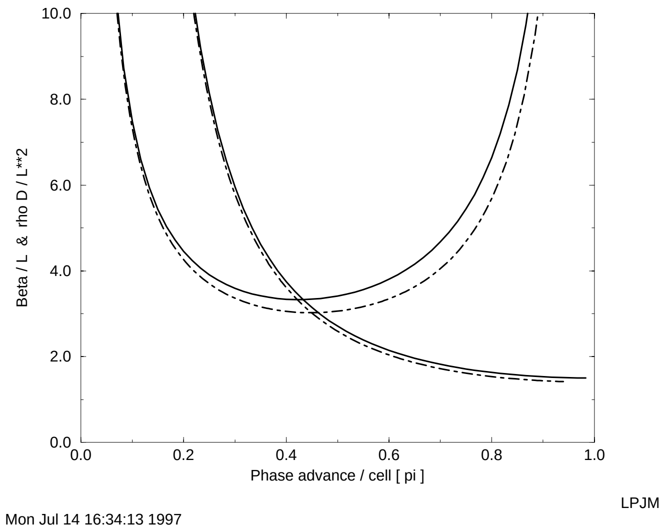

Figure 2: Comparison of β_+/L and $D_+\rho/L^2$ for the separated function FODO and combined function FD models.

The scaling coefficients are tabulated below for the golden cell and for cells with 60° and 90° phase advance.

	60°	Golden Cell	90°
β_+/L	$6/\sqrt{3} = 3.46\ldots$	$1/r_G^{3/2} + 1/r_G^{1/2} = 3.33\ldots$	$2+\sqrt{2} = 3.41\ldots$
β_-/L	$2/\sqrt{3} = 1.15\ldots$	$1/r_G^{3/2} + 1/r_G^{1/2} = 0.78\ldots$	$2-\sqrt{2} = 0.58\ldots$
$N^2\alpha$	$4\pi^2 = 39.47\ldots$	$(\pi/r_G)^2 = 25.83\ldots$	$2\pi^2 = 19.73\ldots$
ξ/N	$-1/\pi\sqrt{3} = -0.18\ldots$	$-r_G^{1/2}/\pi = -0.25\ldots$	$-1/\pi = -0.31\ldots$
$B'l/BN$	$1/\pi = 0.31\ldots$	$2r_G/\pi = 0.39\ldots$	$\sqrt{2}/\pi = 0.45\ldots$

As a perhaps not totally worthless exercise, consider using the curve for $\beta_+/2f$, given by Eq.(2), rather than $\beta_+/2L$. Does one arrive at the same conclusions?

2 Combined function models

Curves analagous to those produced by Eq.(1) and Eq.(3) but for a combined function FD model are shown as the dashed lines in Figure 2; the solid lines are repetitions of the FODO calculations, put here for the sake of comparison. While the scaling properties are no longer exact, there are remarkably small differences when these results are plotted in this way.

However, the pure FD model has no spaces for sextupoles or drifts. If we take, instead, Mishra's lattice for the 3 TeV low field injector [6] and alter the parameters so as to represent a 50 TeV high field cell, the resultant β_+ is plotted in Figure 3, for 1000 cells, and Figure 4, for 500 cells. In the same figures are also plotted the quadrupole gradient required in the bends, in Tesla per meter and as a fraction of the magnetic

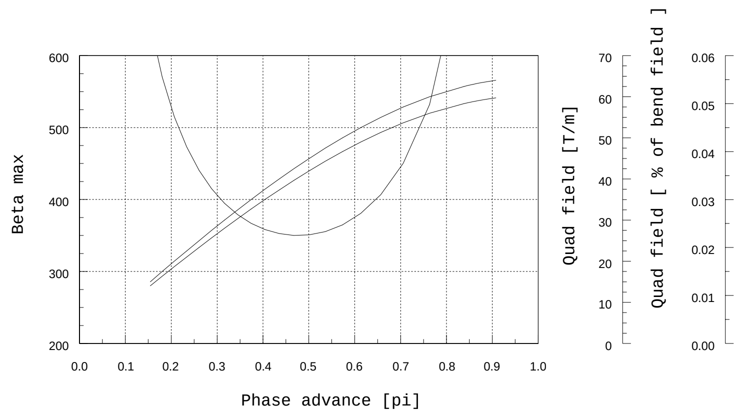

Figure 3: The β_+ curve and required quadupole fields for a combined function lattice with space for sextupole magnets. Here, $N = 1000$.

field at 1 cm from the center. We make only a few observations here. (a) $\min\beta_+$ occurs much closer to $\psi = 90^\circ$ than $\psi = 76^\circ$. (b) The curve is not as flat as before, so that the acceptable domain in ψ is smaller. (c) Most importantly, gradient fields have acceptable values; the gradient field at 1 cm will be $\leq 4\%$ of the bend field.

3 Key question

The key question is: what should be the cell length, or, equivalently, how many standard cells should there be in the ring? The scaling laws of the FODO model – and their approximate counterparts for the combined function models – only underscore the importance of this decision. Larger cells are more economical but will lead to a decreased aperture when nonlinear and error effects are taken into account. As a starting point, and only as a starting point, Glenn Goderre [5] suggested in this workshop using the criterion: (a) $L[\mathrm{m}] \cdot \theta = 0.8$, a rule he obtained from an SSC memo by Courant, *et al.* [2] Don Edwards has privately suggested a similar initial guess: (b) $N = \sqrt{C[\mathrm{ft}]}$. [4] These are, in fact related; (a) implies that $N = 0.77\sqrt{C[\mathrm{ft}]}$, which is remarkably close to (b). If we apply (b) blindly to the 50 TeV machine, we get $N = 522$ and $L = 80$ [m], which are in the right ball park.

At the SSC, 100 m was chosen for the half cell length after considerable study, involving calculating dynamic and linear apertures using many independent tracking programs and checking the results for consistency. As a result of these studies, an empirical power law relating linear aperture to $L^{-3/4}$ was noted and used to fix the 100 m value. (A[mm]$= 180L[\mathrm{m}]^{-0.76}$, for coil diameter of 4 cm.) No corresponding studies were carried out for the combined function model or for either model with the levels of synchrotron radiation expected for the 50 TeV high field machine. (Synchrotron radiation damping and anti-damping

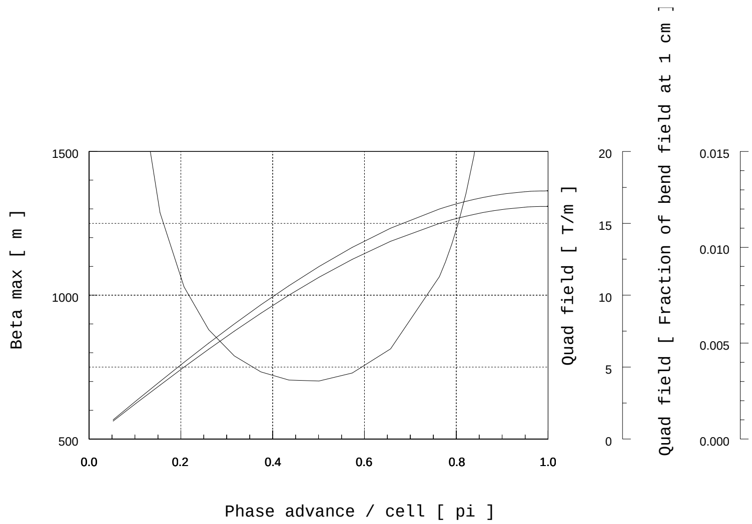

Figure 4: The β_+ curve and required quadupole fields for a combined function lattice with space for sextupole magnets. Here, $N = 500$.

must also be considered in making the choice between a FODO cell and a combined function cell.)

While it is likely that the final answer is still in the neighborhood of 100 m, *I would urge that we take this issue at least as seriously as was done at the SSC.* In particular, this means not making a recommendation after a quickly organized two-week workshop but only after taking the time to understand completely the SSC calculations and to do comparable studies of our own for both models under consideration.

With regard to the "Serenity Prayer," one word is used badly. Rather than "change the things I *can*," a better phrase would have been "change the things I *should*." It is worthwhile keeping that distinction in mind as we proceed to study the VLHC options.

References

[1] Philip J. Bryant and Kjell Johnsen. *The Principles of Circular Accelerators and Storage Rings*. Cambridge University Press, 1993.

[2] E. D. Courant, D. R. Douglas, A. A. Garren, and D. E. Johnson. Ssc test lattice designs. Technical report. SSC Note: SSC-19.

[3] D. A. Edwards and M. J. Syphers. *An Introduction to the Physics of High Energy Accelerators*. John Wiley & Sons, New York, 1993.

[4] Don Edwards. Private communication.

[5] Glenn Goderre. These proceedings.

[6] Shekhar Mishra. These proceedings.